Sabancı University
Faculty of Engineering and Natural Sciences

EE 314 Digital Communications **Spring 2019**

Assoc. Prof. Mehmet Keskinöz

HOMEWORK #2

Assigned: Mar. 01, 2019
Due : Mar. 08, 2019, 4:00 pm

- ***Remarks: Please keep your answers clear and concise and show all the mathematical derivations that you perform. Each student should write up the solutions entirely on their own. You should list your name and ID on your write-up. If you do not type your solutions in a computer, be sure that your hand-writing is legible, your scan is high-quality and your name and ID are clearly written on your submitted document.***
- ***Your solutions should be scanned as a single pdf file (we wont accept other format such as jpeg or multiple files). You should name your pdf file as first_name_lastname_HW_number (e.g., Mehmet_Keskinoz_HW_1)***
- ***If you have a MATLAB problem, you should also be required to submit your .m file (name .m file as first_name_lastname_HW_number and write your name and ID as a commented header in the .m file) .***
- ***if you don't have a MATLAB related problem in your homework, just upload your solutions as a single pdf file . Otherwise, you should zip your .m file together with your pdf file (name the zip file as as first_name_lastname_HW_number) and upload your single zip file to SUCOURSE.***
- ***Note that you can only get help from your TAs on MATLAB related questions during their office hours.***
- ***If you want to get feedbacks about your homework, you should also submit hand-written (or hard-copy) of your solutions.***
- ***Late submission will not be accepted***

(1) Find the average normalized power in the waveform $x(t) = 2 + 5\sin(6\pi t) + 15\cos(12\pi t)$

i) using time averaging
ii) using summation of spectral coefficients.

Solution:

$$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |x(t)|^2 dt = 3\int_{-\frac{1}{6}}^{\frac{1}{6}} \left[4 + 25\sin^2(6\pi t) + 225\cos^2(12\pi t)\right] dt$$

i)

$2\pi f_0 = 6\pi \to T_0 = \frac{1}{3}$

$P_x = 4 + \frac{25}{2} + \frac{225}{2} = \boxed{129W}$

ii) $G_x(f) = \sum_{-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$

by using Inverse Euler's identity:

$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$

$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$

$\Rightarrow c_{-1} = c_1 = \frac{15}{2}$, $c_{-2} = c_2 = \frac{15}{2}$, $c_0 = 2$, $c_n = 0,\ n = \mp 3, \ldots$

$G_x(f) = 4\delta(f) + \frac{25}{4}\delta(f-3) + \frac{25}{4}\delta(f+3) + \frac{225}{4}\delta(f-6) + \frac{225}{4}\delta(f+6)$

$P_x = \int_{-\infty}^{\infty} G_x(f)\,df = 4 + \frac{25}{2} + \frac{225}{2} = \boxed{129W}$

(2) Let $y(t) = x^2(t)$ and $x(t) = 400\,\text{sinc}(400t)$. Determine the followings:

i. Is $y(t)$ an energy or power type signal? Why?

ii. Determine the spectral density of $y(t)$?

iii. Determine bandwidth of $y(t)$.

Solution:

4)

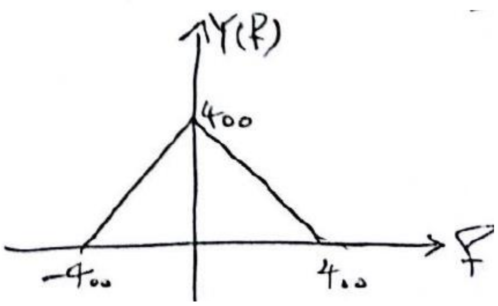
i) $y(t) = x^2(t) = 160000\,\text{sinc}^2(400t)$

$\text{sinc}^2(kt) \xleftrightarrow{F} \frac{1}{k}\Lambda\left(\frac{f}{k}\right)$

$Y(f) = 400\,\Lambda\left(\frac{f}{400}\right)$

$E_y = \int_{-\infty}^{\infty} |Y(f)|^2 df = (400)^2 \int_{-\infty}^{\infty} \Lambda^2\left(\frac{f}{400}\right) df = (400)^2 \int_{-400}^{400} \Lambda^2\left(\frac{f}{400}\right) df < \infty$, So Energy Signal

ii) ESD: $\Psi_y(f) = |Y(f)|^2 = (400)^2 \Lambda^2\left(\frac{f}{400}\right)$



iii) $BW = 400\,Hz$

(3)

A signal $g(t) = \text{sinc}^2(5\pi t)$ is sampled (using uniformly spaced impulses) at a rate of: **(i)** 5 Hz; **(ii)** 10 Hz; **(iii)** 20 Hz. For each of the three case:

(a) Sketch the sampled signal.

(b) Sketch the spectrum of the sampled signal.

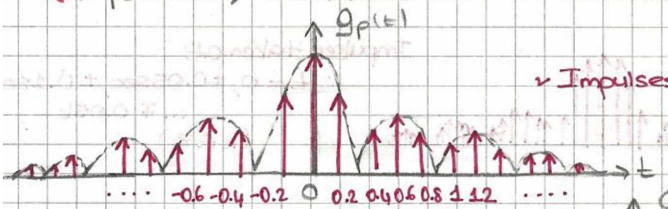
(c) Explain whether you can recover the signal $g(t)$ from the sampled signal.

(d) If the sampled signal is passed through an ideal low-pass filter of bandwidth 5 Hz, sketch the spectrum of the output signal.

Solution: **There was a typo in the question: the function should be** $g(t) = \text{sinc}^2(5t)$

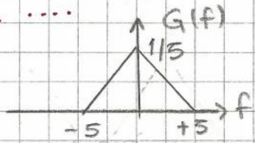
(i) 5Hz

(a) $f_s = 5\text{Hz}$, $T_s = 0.2\text{sec}$

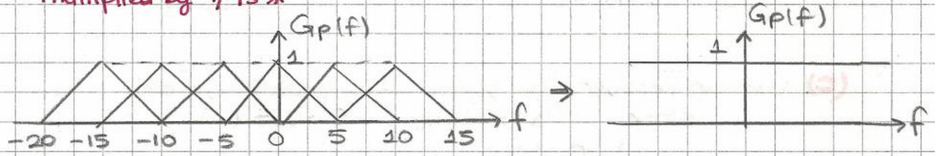


✓ Impulses taken at $t = 0, \pm 0.2, \pm 0.4, \ldots \pm 0.2k$ $(k \in \mathbb{Z})$

(b) $G(f) = \frac{1}{5} \Delta\left(\frac{f}{10}\right)$

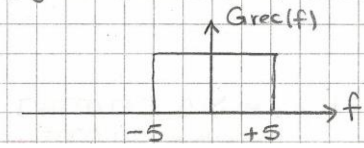


* the periodic extensions will be multiplied by $1/T_s$ *



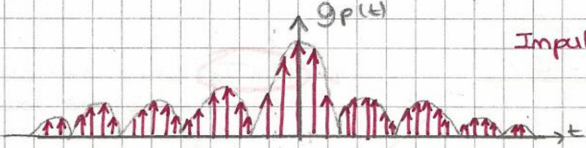
(c) Since there's aliasing (the copies overlap), the original signal is distorted, so it cannot be recovered

(d)



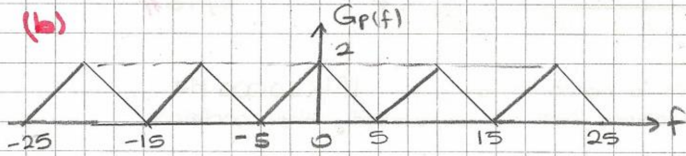
(ii) 10 Hz

(a) $f_s = 10\,Hz$; $T_s = 0.1\,sec$



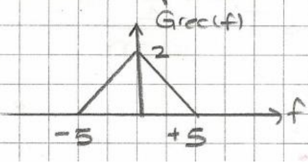
Impulses taken at $t = 0, \pm 0.1, \pm 0.2 \ldots \pm 0.1k$ $(k \in \mathbb{Z})$

(b)



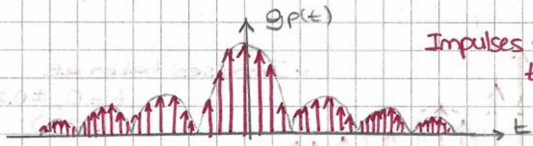
(c) Theoretically, with an <u>ideal</u> LPF, we can recover the signal since the shape is not distorted

(d)



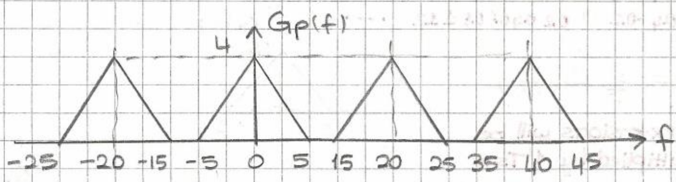
(iii) 20Hz

(a) $f_s = 20\,Hz$; $T_s = 0.05\,sec$



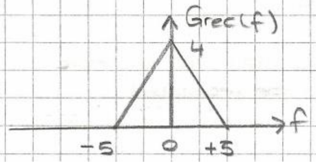
Impulses taken at $t = 0, \pm 0.05\,sec, \pm 0.1\,sec \ldots$
$\ldots \mp 0.05k$
$(k \in \mathbb{Z})$

(b)



(c) We can recover the signal with a practical filter since $\omega_s > 2\omega_m$ $[20 > 2 \times 5]$

(d)



(4)

Using the definition of power-type and energy-type signals,

1. Show that $x(t) = Ae^{j(2\pi f_0 t+\theta)}$ is a power-type signal and its power content is A^2.
2. Show that the unit step signal $u_{-1}(t)$ (the unit step function) is a power-type signal and find its power content.
3. Show that the signal

$$x(t) = \begin{cases} Kt^{-\frac{1}{4}} & t > 0 \\ 0, & t \leq 0 \end{cases}$$

is neither an energy- nor a power-type signal.

Solution:

1)

$$\begin{aligned} P &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| Ae^{j(2\pi f_0 t+\theta)} \right|^2 dt \\ &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A^2\, dt \\ &= A^2 \end{aligned}$$

2)

$$\begin{aligned} P &= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} 1^2\, dt \\ &= \frac{1}{2} \end{aligned}$$

3)

$$\begin{aligned} E &= \lim_{T\to\infty} \int_{0}^{T} K^2/\sqrt{t}\, dt \\ &= \lim_{T\to\infty} \left[2K^2\sqrt{t}\right]_0^T \\ &= \lim_{T\to\infty} 2K^2\sqrt{T} \\ &= \infty \end{aligned}$$

therefore, it is not energy-type. To find the power

$$\begin{aligned} P &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} K^2/\sqrt{t}\, dt \\ &= \lim_{T\to\infty} \frac{1}{2T} 2K^2\sqrt{T} \\ &= 0 \end{aligned}$$

and hence it is not power-type either.