# Name(s): Section: TA:

Applications of Sine and Cosine in Physics

Based on an original exercise by Gillian Galle and Dawn Meredith July 2012 Modified 2021 by kjs

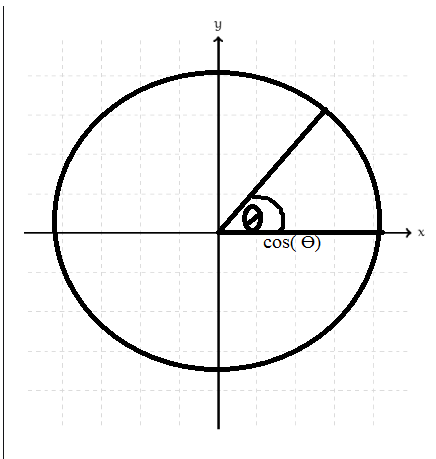
This lab takes trigonometric concepts and put them in a physics context.

# Physical context

Picture a coffee cup sitting on the edge of 15 cm radius turntable in a microwave, and the microwave is running, so the coffee cup is moving in a circle at a constant rate, going around once in 30 seconds. If you get down to eye level with the microwave, you will see just the *x-* component of its motion, and (from our knowledge of the unit circle) *x=*cos*(**).*

1. (**10 point**s) To be sure this makes sense, draw a picture of the turntable from the top and draw

*x-y* axes. Pick one location on the circle and indicate both  and *x* for that location of the cup.



Need for a more refined model

The mathematical model x=cos() is not quite correct or complete. First, we want to include in this description how  changes in time, and also, the turntable is not a unit circle, but a circle of radius 15 cm. In this activity we will learn how to put these refinements in the mathematical model.

How  changes in time.

Recall that for an object moving at a constant rate in a straight line (e.g. a car moving down a straight road) x=x0+vt, where x0 is the location at t=0. Analogously, when the cup moves around

a circle at a constant rate,  = 0 +  t. (We will take 0=0 from now on). The notation  stands for angular velocity.

1. (**5 points**) Recalling linear motion for a moment, if an object moves 30m in 5 seconds, what is v?

**v= d/t=30/5=6m/s**

1. **(15 points**) Using the definition that  = (change in angle)/(change in time), what is  for the cup on the turntable? Be sure to use radians (not degrees) for the angle. ( is conventionally measured in radians/s .)

 =

Using , what angle has the cup traveled to at 15 seconds? Show your work for full credit and be sure to keep units. As a check, what fraction of a cycle is 15 seconds? Does that agree with your angle?

  s =

=

=

Using , what angle has the cup traveled in 600 seconds? Show your work for full credit and be sure to keep units. As a check, how many cycles is 600 seconds? Does that agree with your angle?

 600 s =

=

=5πrad

Using , what angle has the cup traveled to at 15 seconds? Show your work for full credit and be sure to keep units. As a check, what fraction of a cycle is 15 seconds? Does that agree with your angle?

  s =

=

=

Using , what angle has the cup traveled in 600 seconds? Show your work for full credit and be sure to keep units. As a check, how many cycles is 600 seconds? Does that agree with your angle?

 600 s =

=

=5πrad

Verify that the last two equations have correct units for an angle. If not, look for the mistake, or check with the instructor.

1. (20 **points**) We end this task by finding a symbolic way of writing the angular velocity that corresponds to an angular displacement of 2 pi during an unspecified period T seconds.

What is the  in this case?

 =

Using , what angle has the cup traveled in (T/4) seconds?

 =

=

=

Using , what angle has the cup traveled in (T/4) seconds?

 =

=

=

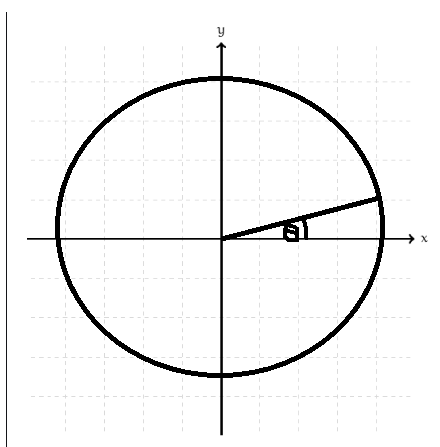
Using , what angle has the cup traveled in (T/4) seconds?

 =

=

=

Now draw the motion along the turntable for the last example. Use this to check that you obtained the correct angle above.



Lastly, using angular velocity, what angle corresponds to *t* seconds, where *t* is an unspecified time? Your answer will have both T and t – these are not the same. “T” stands for a fixed but unspecified time to go around the circle once ; “t” stands for the variable time; think of it as all the various readings of your watch as the cup goes around the turntable.

 t=ωTt

(This is boxed because this general formula will be useful many times as you learn about oscillations and waves.)

1. (**15 points**) Now that we’ve seen how  depends on time for uniform circular motion, let’s see how that works in conjunction with the sine and cosine function. In the table below, “s” stands for seconds.

What is the period of the motion, and how can you tell based on the equations for x(t) in the table?

The period of motion is 5s , to find the period in sine and cause graphs you have to use this equation of period (P=2π/b) where b is x(t)=a\*sine(bt-c)+d

By filling out the “fraction of a cycle” row first, you should be able to fill out the other two rows without a calculator.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *t*  | 0s | 1.25 s | 2.5s | 3.75s | 5s | 6.25s | 7.5s | 8.75s | 10s |
| Fraction of a cycle | 0 | 4/5 | 2/5 | 4/15 | 1/5 | 4/25 | 2/15 | 4/35 | 1/10 |
| *x*(*t*)  sin  2 *t*    (5*s*)     | 0 | 0.027 | 0.055 | 0.082 | 0.113 | 0.141 | 0.164 | 0.190 | 0.217 |
| *x*(*t*)  cos  2 *t*    (5*s*)     | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

# What about the 15 cm radius?

1. (**20 points**) Lastly, we need to take into account that the cup travels on a 15 cm radius turntable, and not the unit circle. Which one of the following mathematical models describes that motion? Choose and then fill out **only** the correct table. Cross out the other three tables, for this problem; consider the case where the coffee completes a revolution in 5 seconds.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *t*  | 0s | 1.25 s | 2.5s | 3.75s | 5s |
| Fraction of a cycle | 0 |  |  |  |  |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *t*  | 0s | 1.25 s | 2.5s | 3.75s | 5s |
| Fraction of a cycle |  |  |  |  |  |
|  |  |  |  |  |  |

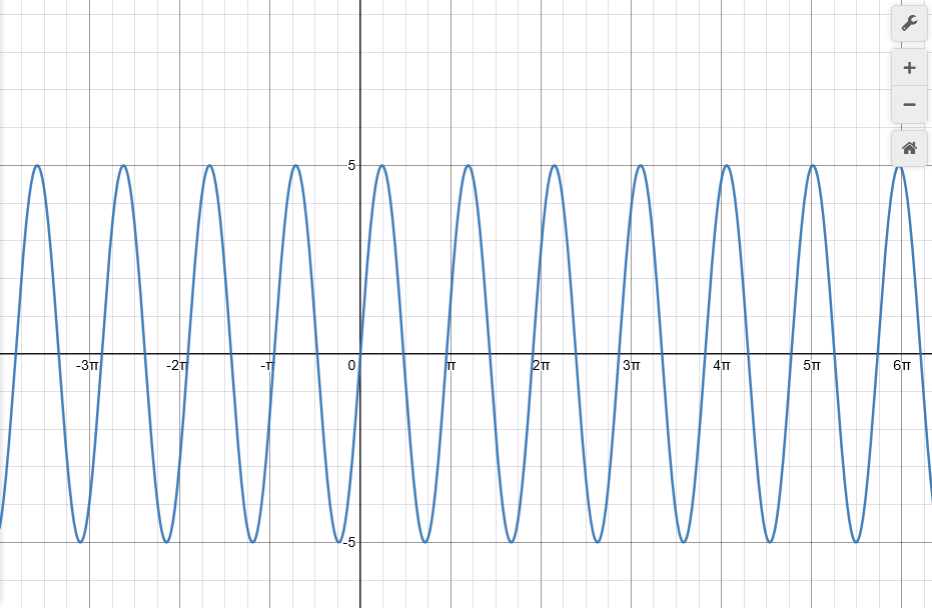
|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *t*  | 0s | 1.25 s | 2.5s | 3.75s | 5s |
| Fraction of a cycle | 0 | 4/5 | 2/5 | 4/15 | 1/5 |
|  | 15 | 15 | 15 | 15 | 15 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *t*  | *t*  | 1.25 s | 2.5s | 3.75s | 5s |
| Fraction of a cycle |  |  |  |  |  |
|  |  |  |  |  |  |

1. (**15 points)** As a check on your understanding, sketch the graphs of the following function. Include two full periods and label the tick marks along both the axes. (In the equations below “*m*” is for meters and “*s*” is for seconds.) Note that these are NOT the position and velocity for the same object. **Use what you learned in today’s activity to make graphing easy; you should not need a calculator**.

(a) position : 

Graph x vs t here.



Period: 3s

Maximum Value: 5m

Minimum Value: -5m

*x*-intercept(s): 0,π,2π,3π,4π,5π,6π