

Solution 1

Given: volume of ice cream pack (V) = $0.2 \times 0.1 \times 0.07 \text{ m}^3$
 $= 1.4 \times 10^{-3} \text{ m}^3$

Initial temperature of ice cream (T_0) = -10°C

Time (t) = 20 minutes
 $= 1200 \text{ seconds}$

Ice cream properties $\rightarrow$

$$K = 2.2 \text{ W/m}^\circ\text{C}$$

$$C = 1800 \text{ J/kg}^\circ\text{C}$$

$$\rho = 900 \text{ kg/m}^3$$

$$h = 10 \text{ W/m}^2^\circ\text{C}$$

Assumptions

- i) $\star$ Internal energy and the temperature of the ice-cream decreases as a result of transient heat conduction.
- ii) Assuming atmospheric temperature (T_∞) as 25°C

Analysis

The Biot number (Bi) is determined from

$$Bi = \frac{h L_c}{K}$$

Here L_c = characteristic length of the ice cream pack

$$= \frac{\text{Volume}}{\text{Surface area}}$$

$$= \frac{1.4 \times 10^{-3}}{2[(0.2)(0.1) + (0.1)(0.07) + (0.07)(0.2)]}$$

$$\Rightarrow L_c = 0.017 \text{ m}$$

$$\Rightarrow Bi = \frac{h L_c}{k} = \frac{10 \times 0.017}{2.2}$$

$$= 0.077$$

Since Biot number is less than 0.1
therefore, lumped analysis is valid

$\Rightarrow$ we can assume the whole ice-cream pack at uniform temperature and write the following relation using energy conservation equation

i.e.
$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = e^{\left(\frac{-h A_s}{\rho V c_p}\right) t}$$

Here, T_i = initial temperature of ice-cream pack = -10°C

T = final temperature of ice-cream pack

T_{∞} = surrounding temperature = 25°C

A_s = surface area of ice cream pack

$\frac{V}{A_s} = L_c$ = characteristic length = 0.017m

c_p = Heat capacity of ice cream =

Putting given values in above relation $\rightarrow$

$$\frac{T - 25}{-10 - 25} = e^{\left(\frac{-10 \times 1200}{900 \times 0.017 \times 1800}\right) t}$$

$$\Rightarrow \frac{T - 25}{-35} = e^{-0.435}$$

$$\Rightarrow T - 25 = (-35)(0.647) = -22.6^{\circ}\text{C}$$

$$\Rightarrow \boxed{T = 2.36^{\circ}\text{C}}$$

Since, the final temperature of the ice cream (i.e. -2.36°C) is above 0°C , it will melt.

And the kids will not eat the ice-cream

Solution 2

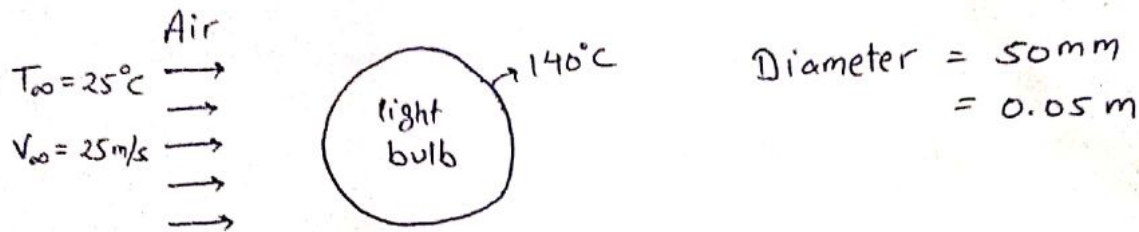


Fig: schematic of the problem

Assumptions

- i) steady operating conditions exist
- ii) Radiation effects are negligible
- iii) Air behaves like an ideal gas
- iv) The outer surface temperature of the light bulb is uniform at all times.
- v) The surface temperature of the light bulb is kept constant.

Properties

The dynamic viscosity of air at the surface temperature is

$$\mu_s = \mu_{@140^{\circ}\text{C}} = 2.345 \times 10^{-5} \text{ kg/m}\cdot\text{s}.$$

The properties of air at the free stream temperature of 25°C

and atmospheric pressure are obtained from (table A-15)

$$k = 0.02551 \text{ W/m}\cdot^{\circ}\text{C}$$

$$\nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\mu = 1.849 \times 10^{-5} \text{ kg/m}\cdot\text{s}$$

$$Pr = 0.7296$$

Reynolds number is determined from

$$\begin{aligned} Re &= \frac{VD}{\nu} \\ &= \frac{(25 \text{ m/s})(0.05 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} \\ &= \underline{\underline{8 \times 10^4}} \end{aligned}$$

The Nusselt number is determined from Whitaker correlation for flow over a sphere $\rightarrow$

$$\begin{aligned} Nu = \frac{hD}{k} &= 2 + \left[0.4 Re^{1/2} + 0.06 Re^{2/3} \right] Pr^{0.4} \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \\ &= 2 + \left[0.4 (8 \times 10^4)^{1/2} + 0.06 (8 \times 10^4)^{2/3} \right] (0.7296)^{0.4} \times \\ &\quad \left(\frac{1.849 \times 10^{-5}}{2.345 \times 10^{-5}} \right)^{1/4} \\ &= 2 + [113.155 + 111.422] (0.881) (0.893) \\ &= \underline{\underline{178.64}} \end{aligned}$$

$\Rightarrow$ the average convection heat transfer coefficient becomes

$$\begin{aligned} h &= \frac{k}{D} Nu \\ &= \frac{0.02551 \text{ W/m}^\circ\text{C}}{0.05 \text{ m}} \times 178.64 \\ &= \underline{\underline{91.145 \text{ W/m}^2 \text{ } ^\circ\text{C}}} \end{aligned}$$

⇒ Heat transfer rate from the light bulb to the surrounding is given by →

$$\dot{Q} = h A_s (T_s - T_{\infty})$$

Here $A_s = \text{surface area} = 4\pi r^2$

$T_s = \text{surface temperature of light bulb} = 140^\circ\text{C}$

$T_{\infty} = \text{surrounding air temperature} = 25^\circ\text{C}$

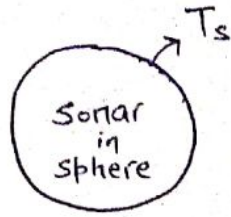
$$\Rightarrow \dot{Q} = (91.145 \text{ W/m}^2\text{ }^\circ\text{C}) \left(4\pi \left(\frac{0.05}{2}\right)^2\right) \text{m}^2 (140 - 25)^\circ\text{C}$$

$$= (91.145) (7.857 \times 10^{-3}) (115) \text{ W}$$

$$= \boxed{82.35 \text{ W}}$$

Solution 3

Water
 $T_{\infty} = 15^{\circ}\text{C}$ $\rightarrow$
 $V_{\infty} = 1\text{ m/s}$ $\rightarrow$
 $\rightarrow$



Diameter = 85 mm
= 0.085 m

$\dot{Q} = 300\text{ W}$

Fig: schematic of the problem

Assumptions

- i) steady operating conditions exist
- ii) Radiation effects are negligible
- iii) Air behaves like an ideal gas
- iv) The outer surface of the sphere has uniform temperature.

Properties of water

The dynamic viscosity of water at the surface temperature is $\mu_s = \mu_{@T_s}$

The properties of water at the free stream temperature of 15°C are as follows

$$k = 0.588\text{ W/mK}$$

$$\gamma = 1.15 \times 10^{-6}\text{ m}^2/\text{s}$$

$$\mu = 1.15 \times 10^{-2}\text{ Ns/m}^2$$

$$Pr = 8.23$$

Reynolds number is given by

$$\begin{aligned} Re &= \frac{VD}{\nu} \\ &= \frac{(1 \text{ m/s})(0.085 \text{ m})}{1.15 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= \underline{\underline{73913}} \end{aligned}$$

Nusselt number is given by

$$\begin{aligned} Nu &= \frac{hD}{K} = 2 + \left[0.4 Re^{1/2} + 0.06 Re^{2/3} \right] \left[Pr^{0.4} \right] \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \\ &= 2 + \left[0.4 (73913)^{1/2} + (0.06) (73913)^{2/3} \right] (8.23)^{0.4} \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \\ &= 2 + (108.74 + 105.67) (2.32) \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \\ &= 2 + 498.22 \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \quad \text{①} \end{aligned}$$

Now, Dynamic viscosity of water at various temperatures $\rightarrow$

$$\mu_{\infty} = \mu_{15^\circ\text{C}} = 1.15 \times 10^{-3}$$

$$\mu_{20^\circ\text{C}} = 1.002 \times 10^{-3}$$

$\Rightarrow$ by interpolation

$$\mu_T = (1.612 - 0.0305 T) \times 10^{-3}$$

$\Rightarrow$ Putting in equation ①

$$Nu = 2 + 498.22 \left(\frac{1.15 \times 10^{-3}}{(1.612 - 0.0305 T_s) \times 10^{-3}} \right)^{1/4}$$

$$\Rightarrow Nu = 2 + 498.22 \times \left(\frac{1.15}{1.612 - 0.0305 T_s} \right)^{1/4}$$

$\Rightarrow$ average convection heat transfer coefficient is given by $\rightarrow$

$$\begin{aligned} h &= \frac{k}{D} Nu \\ &= \left(\frac{0.588}{0.085} \right) \left[2 + 498.22 \times \left(\frac{1.15}{1.612 - 0.0305 T_s} \right)^{1/4} \right] \\ &= 13.835 + 3446.51 \times \left(\frac{1.15}{1.612 - 0.0305 T_s} \right)^{1/4} \end{aligned}$$

given Heat transfer rate $\dot{Q} = 300 \text{ W}$

$$\Rightarrow \dot{Q} = 300 \text{ W} = h A_s (T_s - T_\infty)$$

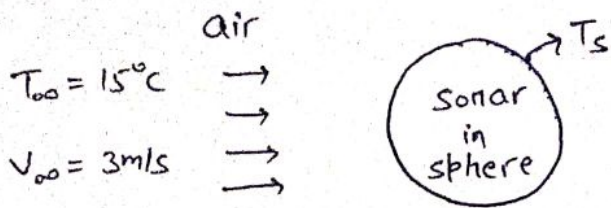
$$\begin{aligned} \Rightarrow 300 &= \left[13.835 + 3446.51 \times \left(\frac{1.15}{1.612 - 0.0305 T_s} \right)^{1/4} \right] \left(4\pi \left(\frac{0.085}{2} \right)^2 \right) \\ &\quad \times (T_s - 15) \end{aligned}$$

$$\Rightarrow 2 \times 6605.85 = (T_s - 15) \left[13.835 + 3446.51 \times \left(\frac{1.15}{1.612 - 0.0305 T_s} \right)^{1/4} \right]$$

By solving above equation, we get

$$\boxed{T_s = 18.89^\circ \text{C}} \quad \text{sonar surface temperature}$$

Now, when the sonar was put into air $\rightarrow$



Properties of air at free stream temperature of $15^{\circ}\text{C} \rightarrow$

$$k = 0.02476 \text{ W/mK}$$

$$\nu = 1.470 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Pr = 0.7323$$

$$\mu = 1.8 \times 10^{-5} \text{ N s/m}^2$$

Reynolds number is given by $\rightarrow$

$$\begin{aligned} Re &= \frac{VD}{\nu} \\ &= \frac{(3\text{ m/s})(0.085\text{ m})}{1.470 \times 10^{-5} \text{ m}^2/\text{s}} \\ &= \underline{\underline{17347}} \end{aligned}$$

Nusselt number is given by

$$\begin{aligned} Nu &= \frac{hD}{k} = 2 + \left[0.4 Re^{1/2} + 0.06 Re^{2/3} \right] (Pr^{0.4}) \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \\ &= 2 + \left[0.4(17347)^{1/2} + 0.06(17347)^{2/3} \right] (0.7323)^{0.4} \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \\ &= 2 + 82 \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \quad (2) \end{aligned}$$

Now, dynamic viscosity of air at various temperatures $\rightarrow$

$$\mu_{0^\circ\text{C}} = \mu_{15^\circ\text{C}} = 1.8 \times 10^{-5}$$

$$\mu_{20^\circ\text{C}} = 1.825 \times 10^{-5}$$

$\Rightarrow$ by interpolation

$$\mu_T = (1.731 + 0.0047 T) \times 10^{-5}$$

Putting in equation (2)

$$Nu = \frac{hD}{K} = 2 + 82 \times \left(\frac{1.8}{1.731 + 0.0047 T} \right)^{1/4}$$

$$\begin{aligned} \Rightarrow h = \frac{K}{D} Nu &= \frac{0.02476}{0.085} \times \left[2 + 82 \times \left(\frac{1.8}{1.731 + 0.0047 T} \right)^{1/4} \right] \\ &= 0.582 + 23.89 \left(\frac{1.8}{1.731 + 0.0047 T} \right)^{1/4} \end{aligned}$$

given, Heat transfer rate $\dot{Q} = 300 \text{ W}$

$$\Rightarrow \dot{Q} = 300 \text{ W} = h A_s (T_s - T_\infty)$$

$$\Rightarrow 300 = \left[0.582 + 23.89 \left(\frac{1.8}{1.731 + 0.0047 T_s} \right)^{1/4} \right] \left(4\pi \left(\frac{0.085}{2} \right)^2 \right) (T_s - 15)$$

$$\Rightarrow 13211.7 = (T_s - 15) \left[0.582 + 23.89 \left(\frac{1.8}{1.731 + 0.0047 T_s} \right)^{1/4} \right]$$

By solving above equation, we get

$$\boxed{T_s = 709.34^\circ\text{C}} \text{ sonar surface temperature}$$

Since, the surface temperature of sphere (i.e. 709.34°C) in air is very high, this is a reason of concern. It may result in temperature stress on sphere material.

Solution 4

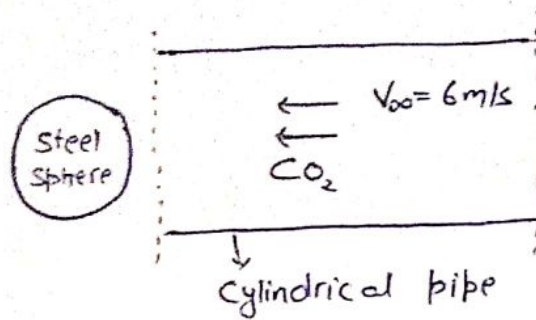


Fig: schematic of the problem

given, diameter of sphere = $d = 5 \text{ cm}$
diameter of pipe = $D = 10 \text{ cm}$

Pipe temperature at entrance = 350 K

Pipe temperature at exit = 550 K

$$\Rightarrow \text{Average temperature of pipe} = \frac{350 \text{ K} + 550 \text{ K}}{2} \\ = 450 \text{ K}$$

gas temperature at entrance = 300 K

gas temperature at exit = 340 K

$$\Rightarrow \text{Average temperature of gas} = \frac{300 \text{ K} + 340 \text{ K}}{2} \\ = 320 \text{ K}$$

Assumptions

- i) Steady operating conditions exist
- ii) Radiation effects are negligible
- iii) CO_2 behaves like an ideal gas

Properties of CO_2 at average temperature (i.e. at 320K) $\rightarrow$

$$K = 0.0396 \text{ W/mK}$$

$$C_p = 1070 \text{ J/kgK}$$

$$\mu = 2.758 \times 10^{-5} \text{ kg/m.s}$$

$$\nu = 3.061 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Pr = 0.7452$$

$$\rho = 0.9636 \text{ kg/m}^3$$

Reynolds number for the flow of CO_2 in the pipe

is given by $\rightarrow$

$$Re = \frac{VD}{\nu}$$
$$= \frac{(6 \text{ m/s})(0.1 \text{ m})}{3.061 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$= \underline{\underline{19601}}$$

Nusselt number for flow through cylindrical pipe is

given by $\rightarrow$

$$Nu = \frac{hD}{K} = 0.193 Re^{0.618} Pr^{1/3} \quad [\text{for } 4,000 < Re < 40,000]$$

$$= (0.193)(19601)^{0.618} (0.7452)^{1/3}$$

$$= (0.193)(449.4)(0.9066)$$

$$= \underline{\underline{78.63}}$$

$\Rightarrow$ convective heat transfer coefficient of the gas moving

in the pipe $(h) = \frac{K}{D} Nu$

$$= \frac{0.0396}{0.1} \times 78.63 = \boxed{31.14 \text{ W/m}^2\text{K}}$$

Now, let the length of the pipe = L

then, by principle of energy conservation $\rightarrow$

Energy gained by the gas = Energy supplied by the pipe
via convection

$$\Rightarrow \dot{m} C_p \Delta T_{\text{gas}} = h A_s \Delta T_{\text{avg}}$$

Here $\dot{m}$ = mass flow rate of the gas

$$= \rho A_{c/s} V_{\infty}$$

$$= (0.9636) \left(\frac{\pi D^2}{4} \right) (6)$$

$$\Rightarrow (0.9636) \left(\frac{\pi D^2}{4} \right) (6) (1070) \Delta T_{\text{gas}} = h (\pi D L) \Delta T_{\text{avg}}$$

$$\Rightarrow 154.66 \Delta T_{\text{gas}} = h L \Delta T_{\text{avg}}$$

ΔT_{gas} = increase in temperature of gas

$$= 340 \text{ K} - 300 \text{ K}$$

$$= 40 \text{ K}$$

ΔT_{avg} = average temperature difference between the
pipe and the gas

$$= 450 \text{ K} - 320 \text{ K}$$

$$= 130 \text{ K}$$

$$\Rightarrow 154.66 \Delta T_{\text{gas}} = h L \Delta T_{\text{avg}}$$

$$\Rightarrow 154.66 \times 40 = (31.14) (L) (130)$$

$$\Rightarrow \boxed{L = 1.53 \text{ m}} \quad \text{Length of pipe}$$

Now, heat transfer rate at the pipe is given by

$$\dot{Q} = h A_s (\Delta T)_{avg}$$

$$= h (\pi D L) \Delta T_{avg}$$

$$= (31.14 \text{ W/m}^2\text{K}) (\pi \times 0.1 \times 1.53 \text{ m}^2) (130 \text{ K})$$

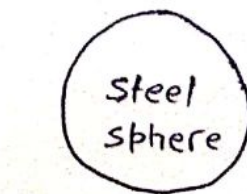
$$= \boxed{1944.3 \text{ W}}$$

Now, interaction between gas and sphere

$$Re = \frac{V d}{\nu}$$

$$= \frac{(6 \text{ m/s}) (0.05)}{3.2564 \times 10^{-5}}$$

$$= \underline{\underline{9213}}$$



$$d = 5 \text{ cm}$$

CO₂ gas

$$\begin{aligned} \leftarrow T_{\infty} &= 340 \text{ K} \\ \leftarrow V_{\infty} &= 6 \text{ m/s} \end{aligned}$$

Properties of gas at 340 K

$$k = 0.0411 \text{ W/mK}$$

$$\mu = 2.833 \times 10^{-5}$$

$$\nu = 3.2564 \times 10^{-5}$$

$$Pr = 0.7453$$

Nusselt number is given by Whitacker correlation

$$\text{i.e. } Nu = \frac{h d}{k} = 2 + \left[0.4 Re^{1/2} + 0.06 Re^{2/3} \right] (Pr^{0.4}) \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4}$$

$$= 2 + \left[0.4 (9213)^{1/2} + 0.06 (9213)^{2/3} \right] (0.7453)^{0.4} \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4}$$

$$= 2 + 57.58 \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4}$$

$$\Rightarrow h = \frac{k}{d} Nu$$

$$= \frac{0.0411}{0.05} \times \left[2 + 57.58 \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \right]$$

$$= 1.644 + 47.33 \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4}$$

Now, Heat transfer rate between sphere and gas is given as $\dot{Q} = 7 \text{ W}$

$$\Rightarrow \dot{Q} = 7 \text{ W} = h A_s (T_s - T_{\infty})$$

T_s = surface temperature of steel sphere

$$\Rightarrow 7 = h (4\pi r^2) (T_s - T_{\infty})$$

$$\Rightarrow 7 = h \left(4\pi \left(\frac{0.05}{2} \right)^2 \right) (T_s - T_{\infty})$$

$$\Rightarrow 890.9 = h (T_s - T_{\infty})$$

$$\Rightarrow 890.9 = \left[1.644 + 47.33 \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \right] (T_s - 340) \quad (1)$$

Now, dynamic viscosities of gas at different temperatures are given by

$$\mu_{\infty} = \mu_{340\text{K}} = 2.833 \times 10^{-5}$$

$$\mu_s = \mu_{@T_s}$$

$$\mu_{320\text{K}} = 2.758 \times 10^{-5}$$

$\Rightarrow$ By interpolation $\rightarrow$

$$\mu_T = (1.545 - 0.0038T) \times 10^{-5}$$

Putting in equation ① $\rightarrow$

$$890.9 = \left[1.644 + 47.33 \times \left(\frac{2.833}{1.545 - 0.0038 T_s} \right)^{1/4} \right] (T_s - 340)$$

By solving above equation, we get

$$\boxed{T_s = 349.7 \text{ K}}$$

surface temperature of sphere