Question 1

$$x = 3-3t \quad \text{—— (1)} \qquad\qquad y = 2t \quad \text{—— (2)}$$

Find value of t from equation (1)

$$x = 3-3t$$

$$x-3 = -3t$$

$$\frac{x-3}{-3} = t \quad \Rightarrow \quad \frac{x}{-3} - \frac{3}{-3} = t$$

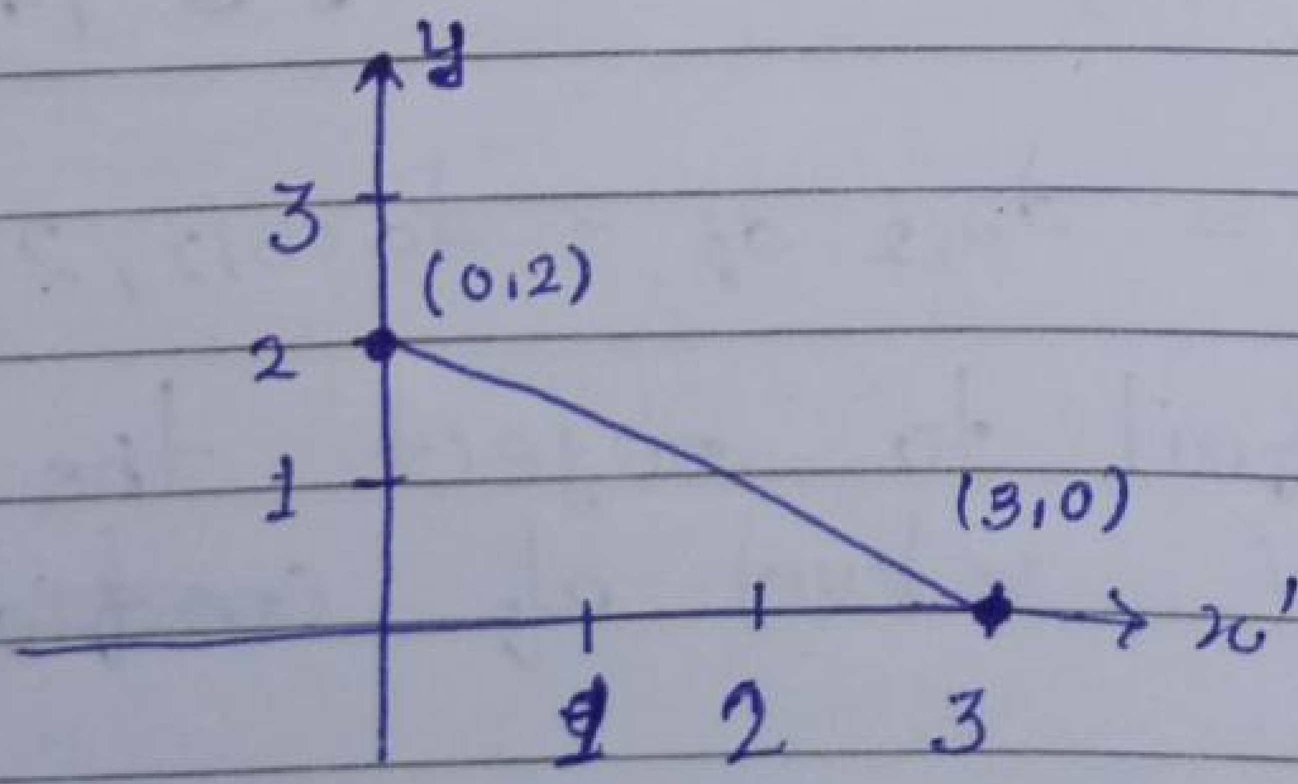
$$1 - \frac{x}{3} = t$$

Put value of t in equation (2)

$$y = 2\left(1 - \frac{x}{3}\right)$$

$$y = -\frac{2}{3}x + 2$$

t	x	y	(x,y)
0	3	0	(3, 0)
1	0	2	(0, 2)



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Question 2

$$x = 4\sin(t) \qquad y = 2\cos(t)$$

$$\frac{dx}{dt} = 4\cos(t) \qquad \frac{dy}{dt} = -2\sin(t)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin(t)}{4\cos(t)} = -\frac{1}{2}\tan(t)$$

Put Value of $t = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{-1}{2}\tan\left(\frac{\pi}{4}\right) = \frac{-1}{2}$$

At $t = \frac{\pi}{4}$ $x = 4\sin(t)$

$$x = 4\sin\frac{\pi}{4} = 4 \times \frac{1}{\sqrt{2}} = 2\sqrt{2}$$

At $t = \frac{\pi}{4}$ $y = 2\cos\frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

So the slope of the tangent at $(2\sqrt{2}, \sqrt{2})$

is $m = \frac{-1}{2}$

We know that

The equation line passing through (x_1, y_1) with the slope m is $y - y_1 = m(x - x_1)$

Here $x_1 = 2\sqrt{2}$ $\quad y_1 = \sqrt{2}$ $\quad m = \frac{-1}{2}$

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$$(y - \sqrt{2}) = \frac{-1}{2}(x - 2\sqrt{2})$$

$$2y - 2\sqrt{2} = -x + 2\sqrt{2}$$

$$x + 2y = 4\sqrt{2}$$

Hence the equation of tangent to the curve at $t = \frac{\pi}{4}$ is

$$x + 2y = 4\sqrt{2}$$

Ques 3 $x = 7t^2 - 9t$, $y = t^6 + 2t^2$

$$\frac{dx}{dt} = 14t - 9 \qquad \frac{dy}{dt} = 6t^5 + 4t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t^5 + 4t}{14t - 9}$$

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$$\frac{d^2y}{dx^2} = \frac{\frac{d(7t^2-9t)}{dt} \times \frac{d^2(t^6+2t^2)}{dt^2} - \frac{d^2(7t^2-9t)}{dt^2} \times \frac{d(t^6+2t^2)}{dt}}{\left(\frac{d(7t^2-9t)}{dt}\right)^3}$$

on Solving

$$\frac{d^2y}{dx^2} = \frac{(14t-9)(30t^4+4) - 14(6t^5+4t)}{(14t-9)^3}$$

Ques 4 The Curve parametric equation given by

$x = \sin(2t)$ $\qquad$ $y = \cos(2t)$

The arc length given by this formula

$$S = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Evaluating the derivatives

$\frac{dx}{dt} = 2\cos(2t)$ $\qquad$ $\frac{dy}{dt} = -2\sin(2t)$

Hence put the value in the arc length formula

$$S = \int_0^{2\pi} \sqrt{(2\cos 2t)^2 + (-2\sin 2t)^2}\, dt$$

$$S = \int_0^{2\pi} \sqrt{4\cos^2 2t + 4\sin^2 2t}\ dt$$

$$S = \int_0^{2\pi} \sqrt{4(\sin^2 2t + \cos^2 2t)}\ dt$$

$$S = \int_0^{2\pi} 2\ dt$$

$$S = 2\int_0^{2\pi} dt$$

$$S = [2t]_0^{2\pi} \Rightarrow 2 \times 2\pi = 4\pi$$

Thus $S = 4\pi$ (arc length)

Ques 5(a)

In this class assignment is not so much hard. I am happy to solve it.

Ques 5(b)

I like Everything in that class.