Question 2

Solution

ⓐ we have it given that:

$B_s(\mathbb{C}^7) = \{e_1, e_2, \dots e_7\}$ is std basis of $\mathbb{C}^7$

where $e_i = (0, \dots 0, 1, 0, \dots 0)$

so:

$M(T, B_s(\mathbb{C}^7))$

$T(e_1) = (4, 0 \dots 0)$

$T(e_2) = (1, 4, 0, \dots 0)$

$T(e_3) = (1, 1, 4, \dots 0)$

$T(e_4) = (1, 1, 1, 4, 0, \dots 0)$

$T(e_5) = (0, 0, 0, 0, 3, 0, 0)$

$T(e_6) = (0, 0, 0, 0, 1, 3, 0)$

$T(e_7) = (0, 0, 0, 0, 1, 1, 3)$

The solution will be:

$$m(T) = \begin{bmatrix} 4 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

(b) Matrix of T is a diagonal matrix and because of this it's eigenvalues are the diagonal entries that we have ~~have~~ at (a) so:

$\{\lambda_k\}_{k=1} = 4, 4, 4, 4, 3, 3, 3$

(c) Eigen space related to $\lambda = 4$
(i)
= Solution set of $[(A-4I)X=0]$ $(A = m(T))$
consider $(A-4I)X=0$

so:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = x_2 = x_1 = 0$

$x_7 = x_6 = x_5 = 0$

Eigenspace corresponding to eigenvalue 4
$= \{(0, 0, 0, x, 0, 0, 0) \mid x \in \mathbb{R}\}$

Considering $(A-3I)x = 0$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_6 = x_5 = 0$

$x_4 = x_3 = x_2 = x_1 = 6$ Eigenspace corresponding to eigenvalue 3
$= \{(0, 0, 0, 0, 0, 0, x) \mid x \in \mathbb{R}\}$

(ii) By jordan cononical form

characteristic polynomial $= (x-4)^4 (x-3)^3$

For finding the minimal polynomial min (m (T)), we can divide matrix into 4 parts

$$
\begin{array}{l}
A_1 \rightarrow \\
\\
m(T) =
\end{array}
\left[\begin{array}{cccc|ccc}
4 & 1 & 1 & 1 & & & \\
0 & 4 & 1 & 1 & & \text{zero} & \\
0 & 0 & 4 & 1 & & \text{matrix} & \\
0 & 0 & 0 & 4 & & & \\
\hline
 & \text{zero} & & & 3 & 1 & 1 \\
 & \text{Matrix} & & & 0 & 3 & 1 \\
 & & & & 0 & 0 & 3
\end{array}\right] \leftarrow A_2
$$

$$
\begin{aligned}
\min(m(T)) &= \operatorname{lcm}(\min(A_1), \min(A_2)) \\
&= \operatorname{lcm}((x-4)^4, (x-3)^3) \\
&= (x-4)^4 (x-3)^3
\end{aligned}
$$

So the jordan form will be:

$$
\text{Jordan form} = \left[\begin{array}{cccc|ccc}
4 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 4 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 3 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 3
\end{array}\right]
$$

1) we have that:

$T(z_1, z_2, z_3, z_4, z_5, z_6, z_7) = (4z_1 + z_2 + z_3 + z_4,\ 4z_2 + z_3 + z_4,$
$4z_3 + z_4;\ 4z_4,\ 3z_5 + z_6 + z_7,\ 3z_7)$

and the standard basis

$(e_1, e_2, e_3, e_4, e_5, e_6, e_7) = \{(1,0,0,0,0,0,0), (0,1,0,0,0,0,0),$
$(0,0,1,0,0,0,0), (0,0,0,1,0,0,0), (0,0,0,0,1,0,0)\ (0,0,0,0,0,1,0)$
$(0,0,0,0,0,0,1)\}$

$T(e_1) = (4,0,0,0,0,0,0)$

$T(e_2) = (1,4,0,0,0,0,0)$

$T(e_3) = (1,1,4,0,0,0)$

$T(e_4) = (1,1,1,4,0,0)$

$T(e_5) = (0,0,0,0,0,3,0)$

$T(e_6) = (0,0,0,0,0,1,0)$

$T(e_7) = (0,0,0,0,0,1,3)$

Now matrix represantation of linear transformation

$$T = \left[\begin{array}{cccc|ccc} 4 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{array}\right]$$

In matrix T we have two block

$$A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

Now matrix of A is upper triangular

diagonal entry are Eigenvalue of A

$\lambda = 4, 4, 4, 4$

Now. $B = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

The characteristic polynomials.

$ch_B(x) = x^3 - 4x^2 + (3+0+0)x - 0 = 0$

$ch_B(x) = x^3 - 4x^2 + 3x$

(e) For any 3×3 matrix A

$ch_A(x) = x^3 - A x^2 + (A_{11} + A_{22} + A_{33})x - \det(A) = 0$

$ch_B(x) = x\,(x^2 - 4x + 3)$

$= x(x^2 - (3+1)x + 3)$

$= x(x^2 - 3x - x + 3$

$= x((x-3)(x-1))$

Eigen value of $B = 0, 3, 1$

Now the T. C form of the matrix is:

$$T = \begin{bmatrix} 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Since $ch_T(x) = (x-4)^4 (x-3)(x-1)(x)$

$M_T(x) = (x-4)^4 (x-3)(x-1)(x)$

Because $(0, 3, 1)$ are distinct Eigenvalue

$ch_B(x) = M_B(x)$

and for $\lambda = 4$

$N(A - \lambda I) \Rightarrow N(A - 4I) = 1$

so: 1 block of order 4

Question 3

Solution

we have given that:

Let $T \in L(V)$ and B be an orthonormal basis so:

$$M(T, B) = \begin{bmatrix} 3 & 1 & & & \\ & 3 & 1 & & \\ & & 3 & & \\ & & & 3 & 1 \\ & & & & 3 \end{bmatrix}$$

we have T that it is the selp adjoints

$T = T^*$

Let $M[T, B] \Rightarrow M^t[T, B]$

$$M^t[T, B] = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \neq M[T, B]$$

so, by here we can say that

T is not a selp- adjoint operation

and T is also not normal

Because:

These are the adjoints into column vectors which it is to be directed

$MM^t \neq M^t M$

It is the first entry MM^t, to be 26 where into $M^t M$ is 25

It is basis ∃ of orthonormal upper triangles be the R. It is true.
We have to self-adjoints and self-normal operators should be initiative of the fields. Through the orthogonal basis of the consulting eigen vectors.
Let us the R has the diagonal table should be but not need net be analysis.
Self-adjoint operators have eigen vectors which are real:

$$\lambda \|v\|^2 = (\dot{v} \cdot \lambda v)$$
$$= \bar{\lambda} \|v\|^2$$
$$\Rightarrow \lambda = \bar{\lambda}$$

It is not to be positive operators have to square roots then R. S. U are the positive vectors in to hold otherwise not.