7.3-1 Let $y = 24$, $n = 642$, and $\alpha = (100-95)\% = 0.05$, so $z_{\alpha/2} = 1.96$.

a $\hat{p} = \frac{y}{n} = \frac{24}{642} = \frac{4}{107}$.

b By 7.3-2,

$$\left[\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] = [0.0227,\ 0.0521]$$

serves as an 95% confidence interval for p.

c By 7.3-4,

$$\frac{\hat{p} + z^2_{\alpha/2}/(2n) - z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n + z^2_{\alpha/2}/(4n^2)}}{1 + z^2_{\alpha/2}/n} = 0.0252.$$

$$\frac{\hat{p} + z^2_{\alpha/2}/(2n) + z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n + z^2_{\alpha/2}/(4n^2)}}{1 + z^2_{\alpha/2}/n} = 0.0550.$$

so, $[0.0252,\ 0.0550]$ serves as an 95% confidence interval for p.

d Calculate $\tilde{p} = \frac{y + z^2_{\alpha/2}/2}{n + z^2_{\alpha/2}} = 0.0401$. By 7.3-5,

$$\left[\tilde{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+z^2_{\alpha/2}}},\ \tilde{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+z^2_{\alpha/2}}}\right] = [0.0250,\ 0.0552].$$

serves as an 95% confidence interval for p.

e We know that $z_\alpha = 1.645$. So, the one-sided 95% confidence interval for p that provides an upper bound for p is

$$\left[0,\ \frac{y}{n} + z_\alpha\sqrt{\frac{\frac{y}{n}\left(1-\frac{y}{n}\right)}{n}}\right] = [0,\ 0.0497].$$

7.3-2 Let $y = 142$, $n = 200$, and $\alpha = (100-90)\% = 0.1$, so $z_{\alpha/2} = 1.645$.

a $\hat{p} = \frac{y}{n} = \frac{142}{200} = 0.71$.

b By 7.3-2,

$$\left[\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] = [0.6572,\ 0.7628].$$

serves as an 90% confidence interval for p.

c By 7.3-4,

$$\frac{\hat{p} + z^2_{\alpha/2}/(2n) - z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n + z^2_{\alpha/2}/(4n^2)}}{1 + z^2_{\alpha/2}/n} = 0.6547$$

$$\frac{\hat{p} + z_{\alpha/2}^2/(2n) + z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n + z_{\alpha/2}^2/(4n^2)}}{1 + z_{\alpha/2}^2/n} = 0.7597$$

so, $[0.6547, 0.7597]$ serves as an 90% confidence interval for p.

d Calculate $\tilde{p} = \dfrac{y + z_{\alpha/2}^2/2}{n + z_{\alpha/2}^2} = 0.7072$. By 7.3-5,

$$\left[\tilde{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n + z_{\alpha/2}^2}},\ \tilde{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n + z_{\alpha/2}^2}}\right] = [0.6546, 0.7598].$$

serves as an 90% confidence interval for p.

e We know that $z_\alpha = 1.28$. So, the one-sided 90% confidence interval for p that provides a lower bound for p is

$$\left[\frac{y}{n} - z_\alpha\sqrt{\frac{\frac{y}{n}\left(1-\frac{y}{n}\right)}{n}},\ 1\right] = [0.6689, 1].$$

7.3-6 We have that $y = 1497$, $n = 5757$, and $\alpha = (100-98)\% = 0.02$, so $z_{\alpha/2} = 2.3263$ and $\hat{p} = \frac{y}{n} = 0.2600$. Then, an approximate 98% confidence interval for p is

$$\left[\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] = [0.2466, 0.2735].$$

7.4-1 We have $\sigma^2 = 4.84$, $\varepsilon = 0.4$, $\alpha = (100-95)\% = 0.05$, so $z_{\alpha/2} = 1.96$.

Then $n = \dfrac{z_{\alpha/2}^2\sigma^2}{\varepsilon^2} = 116.2084$.

Therefore, the sample size needed is 117.

7.4-3 Given $\bar{x} = 6.09$, $s = 0.02$, $\varepsilon = 0.001$, $\alpha = (100-90)\% = 0.1$, so $z_{\frac{\alpha}{2}} = 1.645$.

a The needed sample size is

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{\varepsilon}\right)^2 \approx \left(\frac{z_{\alpha/2}\,s}{\varepsilon}\right)^2 = 1082.41 \approx 1083.$$

b Suppose $n = 1219$, $\bar{x} = 6.048$, $s = 0.022$, $\alpha = (100-90)\% = 0.1$, so $z_{\frac{\alpha}{2}} = 1.645$.

Because $n = 1219$ is large, we can assume $\sigma \approx s = 0.022$.

Therefore, the 90% confidence interval is

$$\left[\bar{x} - z_{\frac{\alpha}{2}}\cdot\frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}}\cdot\frac{\sigma}{\sqrt{n}}\right] = [6.0470, 6.0490].$$

c Calculate $6.09 - 6.048 = 0.042$, it implies the mean has decreased 4.2 times by 0.01 pound. Because \$ 14,000 would be saved per year, then the savings per year with these new adjustments is

$$(4.2)(14.000) = \$58{,}500.$$

d Given $x = 6$, then

$$z = \frac{x-\mu}{\sigma} = \frac{6-6.048}{0.22} = -\frac{24}{11}.$$

Therefore,

$$P(X<6) = P\left(Z< -\frac{24}{11}\right) = P\left(Z>\frac{24}{11}\right) = 1-P\left(Z<\frac{24}{11}\right) = 0.0146.$$

7.4-9 We know that $\varepsilon = 0.02$ and $\alpha = (100-99)\% = 0.01$, so $z_{\frac{\alpha}{2}} = 2.575$.
In a fair six-sided die, we have $\hat{p} = \frac{1}{6}$.
Therefore, the needed sample size is

$$n = \frac{(z_{\frac{\alpha}{2}})^2\,(\hat{p}(1-\hat{p}))}{\varepsilon^2} = 2302.3 \approx 2303.$$